Problem Set on Frictional, Rationing, and Efficient Unemployment

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Problem 1

Consider a matching model with a labor force of size 1. The matching function is Cobb-Douglas:

$$m(U, V) = \sqrt{U \cdot V},$$

where U is the number of unemployed workers and V is the number of vacant jobs. Firms have a production function

$$y(N) = 2 \cdot a \cdot \sqrt{N},$$

where $a \leq 1$ governs labor productivity and N denotes the number of producers in the firm. All workers are paid at a wage

$$w = \sqrt{a}.$$

Firms incur a recruiting cost of $r > 0$ recruiters per vacancy and face a job-destruction rate $s > 0$. The labor market tightness is $\theta = V/U$ and the employment level is $L = 1 - U$.

A) Compute the job-finding rate $f(\theta)$ and vacancy-filling rate $q(\theta)$. Assuming that labor-market flows are balanced, compute the recruiter-producer ratio $\tau(\theta)$. Compute the elasticities of f, q, and τ with respect to θ. Interpret the signs of the elasticities.

B) Assuming that labor-market flows are balanced, compute labor supply $L^s(\theta)$. Compute the elasticity of L^s with respect to θ. Interpret the sign of the elasticity.

C) Firms choose employment to maximize flow profits:

$$y(N) - [1 + \tau(\theta)] \cdot w \cdot N.$$

Compute the labor demand $L^d(\theta, a)$ by solving this maximization problem. Compute the elasticities of L^d with respect to θ and with respect to a. Interpret the signs of these elasticities.

D) Characterize tightness $\theta(a)$ and employment $L(a)$ in the model. Compute the elasticities of $\theta(a)$ and $L(a)$ with respect to a. Interpret the signs of these elasticities.

E) Would shocks to labor productivity a create realistic business cycles?

F) Compute the amount of rationing unemployment $U^r(a)$ and frictional unemployment $U^f(a)$ in the model.

G) Prove that $dU^f/da > 0$. Interpret the result and provide some policy implications.

Problem 2

Consider an economy with a mass 1 of participants in the labor force. The Beveridge curve takes a very simple form:

$$v(u) = \frac{\omega}{u},$$

where $\omega > 0$ governs the location of the Beveridge curve. Each vacancy requires the attention of a full-time worker. Finally, all production takes place in firms and there is no home production at all. As a result, social welfare is determined by the number of producers in firms.

A) Compute the socially efficient labor market tightness $\theta^\star$. How does $\theta^\star$ depend on the parameter ω?

B) Compute the socially efficient unemployment rate $u^\star$ as a function of the actual unemployment and vacancy rates, u and v.

C) Using the formulas derived above, compute the efficient tightness, efficient unemployment rate, and unemployment gap in the United States in December 2021. What are the policy implications of your results?